B.Sc. Part—III Semester—V Examination
MATHEMATICS
(Mathematical Analysis)
Paper—IX

Time : Three Hours] [Maximum Marks : 60

Note :— (1) Question No. **1** is compulsory. Attempt once.

(2) Attempt **ONE** question from each Unit.

1. Choose the correct alternative :

(1) If f, g are bounded functions on [a, b] and P is any partition of [a, b] then L(P, –f) is :

(a) L(P, f) (b) U(P, f)

(c) –U(P, f) (d) –L(P, f)

(2) If P = (1, 3, 4, 5, 6) be the partition of [1, 6] with bounds $m_1 = 1$, $m_2 = 2$, $m_3 = 3$, $m_4 = 4$ then the value of L(P, f) is :

(a) 10 (b) 11

(c) 12 (d) 16

(3) The value of $\overline{|1/2}$ is :

(a) 1 (b) 1/2

(c) π (d) $\sqrt{\pi}$

(4) The value of B(4, 3) is :

(a) $\frac{1}{15}$ (b) $\frac{1}{45}$

(c) $\frac{1}{60}$ (d) 1

(5) Let f(z) = u + iv be analytic and $z = re^{i\theta}$ then C – R equations are :

(a) $u_r = v_\theta,\ u_\theta = -v_r$ (b) $u_r = \frac{1}{r}v_\theta,\ v_r = -\frac{1}{r}u_\theta$

(c) $u_r = rv_\theta,\ u_\theta = -\frac{1}{r}v_r$ (d) $u_r = v_\theta,\ u_\theta = v_r$

(6) If (z) is analytic function with constant modulus then f(z) is :

(a) Unbounded (b) Zero

(c) Constant (d) None

(7) The fixed points of the bilinear transformation $w = \frac{z-1}{z+1}$ are :

(a) 1, –1 (b) i, –i

(c) 0, 1 (d) 1, 2

(8) The normal form of bilinear transformation having two finite fixed points p, q is :

(a) $\frac{w-p}{w-q} = k\frac{z-p}{z-q}$ (b) $\frac{w+p}{w+q} = k\frac{z+p}{z+q}$

(c) $\frac{w-p}{w+q} = k\frac{z-p}{z-q}$ (d) $\frac{w-p}{w-q} = k\frac{z+p}{z+q}$

(9) Any finite collection of open set is :

(a) Closed (b) Open

(c) Semi open (d) None

(10) Let A be a subset of metric space X if A is closed then :

(a) A^C is closed (b) A^C is open

(c) $A^C = \phi$ (d) $A^C \neq \phi$ 1×10=10

UNIT—I

2. (a) Prove that a bounded function f defined on [a, b] is integrable on [a, b] iff for each $\in > 0$ there exist a partition P of [a, b] such that $U(P, f) - L(P, f) < \in$. 5

(b) If f is continuous on [a, b] and $|f(x)| \leq k$ for all $x \in [a, b]$ where k is constant, then prove that :

$$\left|\int_a^b f(x)\,dx\right| \leq k(b-a)$$ 5

3. (p) Prove that if f be a bounded and integrable function on [a, b] with m, M as infimum, supremum respectively then there exist a number μ between m and M such that :

$$\int_a^b f(x)\,dx = \mu(b-a)$$ 5

(q) Prove that $\frac{2}{17} < \int_{-1}^{2} \frac{x}{1+x^4}\,dx < \frac{1}{2}$. 5

UNIT—II

4. (a) Let $0 \leq f(x) \leq g(x)$ and $f(x), g(x) \in c$, $a < x < \infty$, then prove the following :

(i) $\int_a^\infty g(x)\,dx < \infty \Rightarrow \int_a^\infty f(x)\,dx < \infty$

(ii) $\int_a^\infty f(x)\,dx = \infty \Rightarrow \int_a^\infty g(x)\,dx = \infty$ 4

(b) Test the convergence or divergence of $\int_{2}^{\infty} \frac{dx}{\sqrt{x^2 - 1}}$. 3

(c) Test the convergence of $\int_{1}^{\infty} \frac{e^{-x}}{x} dx$. 3

5. (p) Prove that $\int_{a}^{\infty} \frac{dx}{x^p}$ converges if $P > 1$ and diverges if $P \le 1$ and $a > 0$. 4

(q) Prove that $B(m, n) = 2 \int_{0}^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$. 3

(r) Evaluate $\int_{0}^{\infty} \frac{dx}{\sqrt{x}\,(1 + x)}$. 3

UNIT—III

6. (a) A necessary condition that $f(z) = u(x, y) + iv(x, y)$ be analytic in a region D is that $u_x = v_y$ and $u_y = -v_x$ in D. 5

(b) Show that $f(z) = \log z$ is analytic and find its derivative. 5

7. (p) If f(z) is an analytic function then prove that :

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2$$ 5

(q) Prove that the function sin z is analytic and find its derivative. 5

UNIT—IV

8. (a) Prove that the cross-ratio remains invariant under a bilinear transformation. 5

(b) Show that the transformation $w = \frac{2z + 3}{z - 4}$ maps the circle $x^2 + y^2 - 4x = 0$ into a straight line $4u + 3 = 0$. 5

9. (p) Under the transformation $w = \frac{1}{z}$ find the image of the circle $|z - 2i| = 2$. 5

(q) Find the bilinear transformation which maps the points $z = 1, i, -i$ into the points $w = 0, 1, \infty$. 5

UNIT—V

10. (a) Let (X, d) be a metric space and $x, y, x', y' \in X$. Show that :

$$|d(x, y) - d(x', y')| \le d(x, x') + d(y, y')$$ 5

(b) If P is a limit point of a set A, then prove that every neighbourhood of P contains infinitely many points of A. 5

11. (p) Define Cauchy sequence and prove that every convergent sequence in a metric space is a Cauchy sequence. 5

(q) Let X be a metric space. If $\{x_n\}$, $\{y_n\}$ are sequence in X such that $x_n \to x$, $y_n \to y$ then show that $d(x_n, y_n) \to d(x, y)$. 5